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Application of numerical methods to study the effect of axial conduction in plates and flow channels on the performance of plate heat exchangers

Received July 2004
Revised November 2004
Accepted February 2005

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Abstract

Purpose – The purpose of this paper is to predict the plate heat exchanger performance when axial conduction in plates and in flow channels are present and fluids' viscosities are temperature dependent.

Design/methodology/approach – The approach to achieve the objective of the paper is deriving the governing equations and developing a computer program based on finite differences to solve them. The governing equations become dimensionless defining reference values and then discretized using FTBCS and FTCS methods. To solve the governing equations, the flow channel is divided into small elements in axial direction. Physical properties are constant for each element, while viscosity changes from one element to another one.

Findings – The effect of axial conduction in plates as well as in flow channels on temperature distributions, are studied individually and simultaneously. The program is run under four different conditions, namely: no axial conduction, axial conduction in the plates and in the flow channels, axial conduction in the plates only, and axial conduction in the flow channels only.

In all the above cases, temperature distributions are achieved and characteristic curves are plotted. The numerical results are validated by comparing them with those published in an established reference carried out ignoring the effect of axial conduction, using the same plate geometry and flow details.

Originality/value – This paper gives valuable information and offers practical help to plate heat exchanger design engineer in order to choose the proper material for the plates as well as the right service and product fluids.

Keywords Heat conduction, Finite difference methods, Numerical analysis, Heat exchangers

Paper type Research paper

Nomenclature

A_x	= channel cross section area
A_{xp}	= plate cross section area
b	= mean channel gap, the internal height of the corrugation
c^*	= heat capacity rate ratio
C	= heat capacity
$_c$	= subscript for cold fluid
c	= specific heat

International Journal of Numerical Methods for Heat & Fluid Flow
Vol. 16 No. 1, 2006
pp. 67-83

0961-5539
DOI 10.1108/09615530610636964

h	= heat transfer coefficient	T	= temperature
$_h$	= subscript for hot fluid	t	= time
i	= axial section designation	U	= local overall heat transfer coefficient
j	= channel and plate designation as shown in Figure 2	$\bar{U}$	= mean overall heat transfer coefficient
k	= thermal conductivity of fluid	u	= flow velocity
L	= plate length	W	= flow channel width
m	= number of flow channels	x	= coordinate along the plates, axial direction shown in Figure 2
$_{min}$	= subscript for minimum	υ	= dimensionless velocity
n	= flow direction designation	ε	= heat exchanger effectiveness
NTU	= number of transfer units	ρ	= fluid density
$_p$	= subscript for plate	τ	= dimensionless time
r	= exponent in		

Introduction

Plate heat exchangers are a relatively new generation of heat exchangers being widely used in food and medicine processing, chemical and petrochemical plants, cryogenic and refrigeration industries and power generating stations. The major advantage of plate heat exchangers is accessibility to the internal surfaces for manual cleaning of these surfaces. This advantage has placed the plate heat exchangers the focus of interest in food and medicine producing factories. Considerable improvements in performance and operating characteristics of plate heat exchangers have been achieved since Seligman (1964) was building the first plate heat exchanger in the early 1920s. As a result of this, plate heat exchangers are nowadays used in an extensive range of heat transfer applications in different industries, so that for liquid to liquid heat transfer at low and medium pressures they are becoming a substitute for traditional shell and tube heat exchangers.

Corrugated plate geometry makes the flow in the channels of a plate heat exchanger very complicated. A wide range of research activities has been done in the last four decades and is still carried out to understand the pressure drop characteristics and thermal performance of plate heat exchangers experimentally (Heggs *et al.*, 1997; Wright and Heggs, 2002a). Modern high speed computers having large storage capacities and improved numerical schemes, being truly the two wings of computational fluid dynamics (CFD), have played a considerable contribution in advancement and fulfillment of these research activities (Ciofalo *et al.*, 1996).

Many simulation programs have been published to predict the pressure drop and heat transfer characteristics of plate heat exchangers (Mehrabian and Poulter, 2000; Sharifi *et al.*, 1995; Wright and Heggs, 2002b; Gut *et al.*, 2004). A number of simplifying assumptions have been used in any of these programs to ease mathematical modeling. These assumptions vary from one program to another, making the numerical results deviate from real experimental measurements. Among these simplifications, the most important ones are as follows:

- physical properties of flow are constant;
- overall heat transfer coefficient is constant;
- effect of corrugation angle on flow phenomena are ignored;
- flow mal-distribution effects are ignored;
- axial conduction in plates and flow channels are ignored;

- steady state condition is assumed;
- entrance effects on flow distribution are ignored; and
- countercurrent flow arrangement prevails.

It is evident, however, taking into account the effect of any of these parameters makes the simulation process very complicated. For example, Mehrabian and Poulter (2000) looked at the effect of corrugation angle on the hydrodynamics and thermal characteristics of plate heat exchangers. They showed that the inclination angle between the plate corrugation and the overall flow direction is a major parameter in the thermo-hydraulic performance of plate heat exchangers. A change in corrugation angle affects the basic flow structure which the primary factor influencing the pressure drop and heat transfer rate. For a given imposed pressure drop, the Reynolds numbers decrease and friction factors increase as corrugation angle increases.

Sharifi *et al.* (1995) studied the dynamic response of plate heat exchangers to a step input of the hot stream temperature and compared their results with the steady state temperature profiles. They also validated their numerical results by comparing them to experimental results obtained in a test rig using specially designed flat plates. Wright and Heggs (2002b) analyzed the performance of plate heat exchangers when one stream undergoes a phase change, especially condensation. Their analysis was then extended to systems in which the heat transfer correlation is dependent upon the quality of the phase change stream. They finally developed a simple methodology to predict the performance of plate heat exchangers under the assumption of constant overall heat transfer coefficient in either co-current or countercurrent arrangements.

Gut *et al.* (2004) carried out a simulation program focusing on the effect of flow mal-distribution inside the channels on the heat transfer performance of plate heat exchangers with non-ideal configuration.

Khoramabadi (2004) has conducted a simulation program to predict one dimensional temperature distribution of flow in the channels of a plate heat exchanger when the fluids' viscosities vary with respect to temperature. He validated his finite difference-based computer program by incorporating the plate dimensions and flow details used to obtain experimental data in Haseler *et al.* (1992) and achieved quantitative agreement between numerical and experimental results.

Mehrabian (2003) developed an analytical-numerical approach to work out the longitudinal temperature changes of flow in the passages of a plate heat exchanger. Uniform heat flux, constant overall heat transfer coefficient, linear relationship between U and T, and linear relationship between U and ΔT were four special cases for which he solved the system of coupled, simultaneous differential equations obtained from the energy balance equation for the control volumes in the hot and cold stream channels.

Mehrabian *et al.* (1997) studied the effect of herringbone angle on the performance of plate heat exchangers by modeling a small repeatable segment of the exchanger channel. This segment is between two single corrugated plates bounded by four contact points, arranged in a way that the plates' principle axes intersect at twice the corrugation angle.

Rao *et al.* (2002) studied the effect of flow distribution to the channels on the thermal performance of a plate heat exchanger. They brought out the influence of flow mal-distribution from channel to channel comprehensively. They indicated the importance of considering the heat transfer coefficient inside the channels as a function of flow rate through that particular channel. They recommended that determination of the heat transfer coefficient should be done by solving an inverse problem using experimental values of temperature and an usual definition of NTU to obtain the value for exponent of Reynolds number in heat transfer correlation by matching with computed plots. This eliminates the entry of the flow distribution effect into the heat transfer data.

Kroger (1967) studied the effect of axial conduction in plates on thermal performance of plate heat exchangers with counter-flow arrangement. He suggested that axial conduction in plates has the most severe influence on exchanger's thermal performance when the heat capacity rate ratio (c^*) is equal to one. He also developed relationships to predict the performance of plate heat exchangers with respect to dimensionless parameter (λ) representing the axial conduction in plates. These relationships apply only for $c^* \leq 1$ and for a limited range of NTU.

Sharifi *et al.* (1995) analyzed a plate heat exchanger with counter-flow arrangement in transient and steady state conditions, numerically. They used different numerical methods to predict the temperature distribution in steady state condition as well as fluid temperatures at exit of flow channels in transient condition. They compared the latter temperatures with those obtained from experimental measurements. In this numerical analysis, they ignored the axial conduction in plates and flow channels.

This paper aims to predict the plate heat exchanger performance when axial conduction in plates and in flow channel are present and fluids' viscosities are temperature dependent. The plate dimensions and flow details used to obtain numerical results in Sharifi *et al.* (1995) are incorporated in the present computer program in order to make a reasonable comparison between the two sets of results. The effect of axial conduction in plates and in flow channels has been highlighted when such comparison is performed.

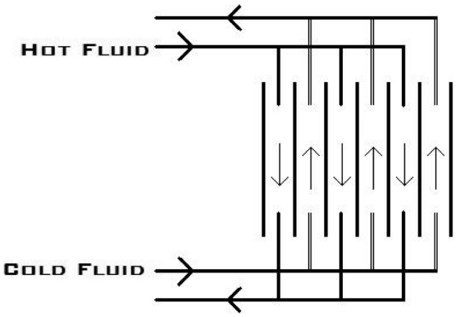
Mathematical modeling

In this study, a counter-flow plate heat exchanger with U-shape configuration similar to that used in Sharifi *et al.* (1995) has been analyzed. Seven stainless steel (SS-304) flat plates make six flow channels. The configuration of the exchanger is shown in Figure 1.

The mathematical modeling for plates and flow channels is developed using the energy balance equation and taking into account the following assumptions:

- the two side channels are insulated;
- heat loss to the ambient is ignored;
- no phase change (evaporation and condensation) occurs;
- physical properties except viscosity are constant;
- plug (one-dimensional) flow prevails; and
- hot and cold fluids are uniformly distributed between the corresponding flow channels.

The uniform flow assumption is appropriate, because the mean channel gap is very small, the plates are assumed to be flat and the flow in plate heat exchanger channels is turbulent even at low Reynolds numbers. Thus, the energy balance equation for the control volume designated in the jth channel of the heat exchanger shown in Figure 2 is written as:



Notes: The hot fluid flows downwards in the odd-numbered channels, while the cold fluid flows upwards in the even-numbered channels. The flow arrangement is U-type, counter-current (looped)

Figure 1.
Configuration of plate heat exchanger with 7 flat plates and 6 flow channels

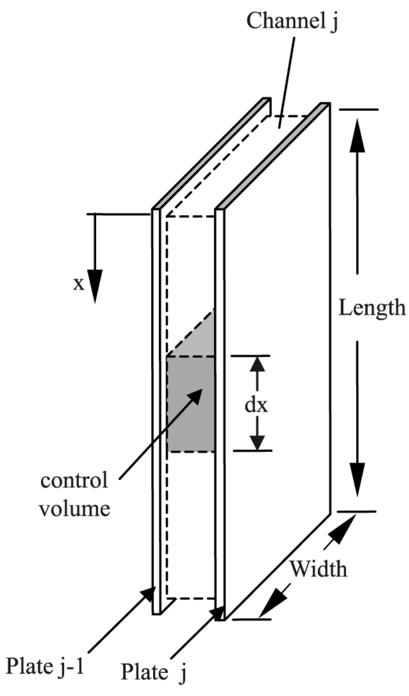


Figure 2.
Flow channel bounded by two plates forming the thermal control volume

$$h_j W \mathrm{d}x(T_{P_{j-1}} - T_j) + h_j W \mathrm{d}x(T_{P_j} - T_j) + A_\mathrm{x} k_j \mathrm{d}x \frac{\partial^2 T_j}{\partial x^2}$$
$$= \rho_j A_\mathrm{x} c_j \mathrm{d}x \left(\frac{\partial T_j}{\partial t} + n_j u_j \frac{\partial T_j}{\partial x} \right) \tag{1}$$
$$(j = 1, \ldots, m)$$

where, the physical meaning of each term in equation (1) from left to right is:

- heat transfer rate from plate $j-1$ to flow in channel j;
- heat transfer rate from plate j to flow in channel j;
- heat conduction rate in flow in channel j in x direction;
- rate of change of internal energy of flow in channel j; and
- heat convection rate in flow in channel j in x direction.

The energy equation for the jth plate as shown in Figure 2 is expressed as:

$$h_j W \mathrm{d}x(T_j - T_{P_j}) + h_{j+1} W \mathrm{d}x(T_{j+1} - T_{P_j}) + A_\mathrm{xp} k_\mathrm{p} \mathrm{d}x \frac{\partial^2 T_{P_j}}{\partial x^2} = \rho_\mathrm{p} A_\mathrm{xp} c_\mathrm{p} \mathrm{d}x \frac{\partial T_{P_j}}{\partial t} \tag{2}$$

where, the physical interpretation of each term in equation (2) from left to right is:

- heat transfer rate from flow in channel j to plate j;
- heat transfer rate from flow in channel $j+1$ to plate j;
- heat conduction rate in plate j in x direction; and
- rate of change of internal energy of plate j.

Assuming the side plates being insulated, the energy equations for the side channels ($j = 1$ and $j = m$) are derived modifying equation (1),

$$h_1 W \mathrm{d}x(T_{P_1} - T_1) + A_\mathrm{x} k_1 \mathrm{d}x \frac{\partial^2 T_1}{\partial x^2} = \rho_1 A_\mathrm{x} c_1 \mathrm{d}x \left(\frac{\partial T_1}{\partial t} + n_1 u_1 \frac{\partial T_1}{\partial x} \right) \tag{3}$$

$$h_m W \mathrm{d}x(T_{P_{m-1}} - T_m) + A_\mathrm{x} k_m \mathrm{d}x \frac{\partial^2 T_m}{\partial x^2} = \rho_m A_\mathrm{x} c_m \mathrm{d}x \left(\frac{\partial T_m}{\partial t} + n_m u_m \frac{\partial T_m}{\partial x} \right) \tag{4}$$

In the above equations, W is the plate width, dx the control volume length, A_x the channel cross sectional area, A_xp the plate cross sectional area, c the fluid specific heat, k the fluid thermal conductivity, k_p the plate thermal conductivity, T the temperature, t the time, u the fluid velocity and ρ the fluid density. Subscript n represents the flow direction, $n = 1$ for flow in the positive x direction while $n = -1$ for flow in the negative x direction. Equations (1)-(4) become dimensionless using the following dimensionless parameters:

$$X = x/L, \quad \theta = \frac{T - T_\mathrm{c,in}}{T_\mathrm{h,in} - T_\mathrm{c,in}}, \quad \upsilon = \frac{u}{u_f}, \quad \tau = t/t_R, \quad t_R = \frac{L}{u_f}$$

In which, L is the channel length, $T_{c,in}$ the cold fluid inlet temperature, $T_{h,in}$ the hot fluid inlet temperature, u_f maximum fluid velocity, X dimensionless distance from the top of the plate, θ the dimensionless temperature, υ the dimensionless velocity and τ the dimensionless time.

Introducing the dimensionless parameters into equations (1)-(4) gives:

$$\frac{\partial \theta_j}{\partial \tau} + n_j \upsilon_j \frac{\partial \theta_j}{\partial X} = a_j(\theta_{P_{j-1}} + \theta_{P_j} - 2\theta_j) + d_j \frac{\partial^2 \theta_j}{\partial X^2} \tag{5}$$

$$\frac{\partial \theta_{P_j}}{\partial \tau} = b_j(\theta_j - \theta_{P_j}) + b_{j+1}(\theta_{j+1} - \theta_{P_j}) + e \frac{\partial^2 \theta_{P_j}}{\partial X^2} \tag{6}$$

$$\frac{\partial \theta_1}{\partial \tau} + n_1 \upsilon_1 \frac{\partial \theta_1}{\partial X} = a_1(\theta_{P_1} - \theta_1) + d_1 \frac{\partial^2 \theta_1}{\partial X^2} \tag{7}$$

$$\frac{\partial \theta_m}{\partial \tau} + n_m \upsilon_m \frac{\partial \theta_m}{\partial X} = a_m(\theta_{P_{m-1}} - \theta_m) + d_m \frac{\partial^2 \theta_m}{\partial X^2} \tag{8}$$

in which a_j, b_j, e and d_j are defined as:

$$a_j = \frac{h_j WL}{\rho_j c_j A_x u_f}, \quad b_j = \frac{h_j WL}{\rho_p c_p A_{xp} u_f}, \quad e = \frac{k_p}{\rho_p c_p L u_f}, \quad d_j = \frac{k_j}{\rho_j c_j L u_f}$$

Equations (5)-(8) are solved using finite difference technique to give temperature distributions in flow channels as well as in plates.

Numerical analysis

To solve the governing equations (5)-(8), the flow channel is divided into small elements each having length Δx and designated by subscript i in x direction. Physical properties are constant for each element, while viscosity changes from one element to another one.

Equation (5) is discretized using forward in time back-central in space (FTBCS) explicit method of the third order (Hoffmann and Chiang, 1993). For the hot fluid channels ($n = 1$), backward difference of the third order is used to approximate the convective term:

$$\frac{\theta_{i,j}^{K+1} - \theta_{i,j}^{K}}{\Delta \tau} + n_j \upsilon_j \left[\frac{\theta_{i+1,j}^{K} - \theta_{i-1,j}^{K}}{2\Delta X} - \frac{\theta_{i+1,j}^{K} - 3\theta_{i,j}^{K} + 3\theta_{i-1,j}^{K} - \theta_{i-2,j}^{K}}{6\Delta X} \right] =$$
$$f_j \frac{\theta_{i+1,j}^{K} - 2\theta_{i,j}^{K} + \theta_{i-1,j}^{K}}{(\Delta X)^2} + a_{i,j}(\theta_{P_{i,j-1}}^{K} + \theta_{P_{i,j}}^{K} - 2\theta_{i,j}^{K}) \tag{9}$$

$$(j = 1, 3, \ldots, 2n - 1, \ldots), \quad (i = 1, 2, 3, \ldots)$$

In Equation (9) the time derivative is expressed in terms of time increments K and $K + 1$ (for-ward difference of the first order), the first space derivative is expressed in terms of grid points $i-2, i-1, i, i+1$ (back-central difference of the third order) and second space derivative is expressed in terms of grid points $i-1$, i, $i+1$ (central difference of the second order).

For the cold fluid channels ($n=-1$), forward difference of the third order is used to approximate the convective term:

$$\frac{\theta_{i,j}^{K+1}-\theta_{i,j}^{K}}{\Delta\tau}+n_j v_j\left[\frac{\theta_{i+1,j}^{K}-\theta_{i-1,j}^{K}}{2\Delta X}-\frac{\theta_{i+2,j}^{K}-3\theta_{i+1,j}^{K}+3\theta_{i,j}^{K}-\theta_{i-1,j}^{K}}{6\Delta X}\right]= \\ f_j\frac{\theta_{i+1,j}^{K}-2\theta_{i,j}^{K}+\theta_{i-1,j}^{K}}{(\Delta X)^2}+a_{i,j}(\theta_{P_{i,j-1}}^{K}+\theta_{P_{i,j}}^{K}-2\theta_{i,j}^{K}) \tag{10}$$

$$(j=2,4,\ldots,2n,\ldots),\quad (i=1,2,3,\ldots)$$

Equation (6) is discretized using forward in time central in space (FTCS) or Euler's explicit method (Hoffmann and Chiang, 1993), giving:

$$\frac{\theta_{P_{i,j}}^{K+1}-\theta_{P_{i,j}}^{K}}{\Delta\tau}=g\frac{\theta_{P_{i+1,j}}^{K}-2\theta_{P_{i,j}}^{K}+\theta_{P_{i-1,j}}^{K}}{(\Delta X)^2}+b_{i,j}(\theta_{i,j}^{K}-\theta_{P_{i,j}}^{K})+b_{i,j+1}(\theta_{i,j+1}^{K}-\theta_{P_{i,j}}^{K}) \tag{11}$$

$$(j=1,2,3,\ldots),\quad (i=1,2,3,\ldots)$$

Equations (5) and (6) have been discretized using FTBCS and FTCS explicit methods, respectively. In the numerical methods mentioned above, the time derivatives are approximated by forward differences, while the second order spatial derivatives are approximated by central differences. Superscript K in equations (9)-(11) designates the time interval and the constants are defined as follows:

$$a_{i,j}=\frac{h_{i,j}W\Delta x}{\rho_j c_j A_{\mathrm{x}} u_j},\quad b_{i,j}=\frac{h_{i,j}W\Delta x}{\rho_{\mathrm{p}} c_{\mathrm{p}} A_{\mathrm{xp}} u_f}\quad g=\frac{k_{\mathrm{p}}}{\rho_{\mathrm{p}} c_{\mathrm{p}}\Delta x u_f},\quad f_j=\frac{k_j}{\rho_j c_j\Delta x u_j}$$

The dimensionless heat transfer coefficient for either stream is obtained from the following empirical correlation:

$$Nu=sRe^{q}Pr^{r} \tag{12}$$

in which, Nu, Re and Pr are the dimensionless Nusselt, Reynolds and Prandtl numbers. Constants s, q and r have been deduced from experimental data (Sharifi *et al.*, 1995) for the heat exchanger with flat plates shown in Figure 1, they are:

$$s=1.35,\quad q=0.65,\quad r=0.33$$

The difference equations (9)-(11) are solved using the following basic assumptions:

- the system initial temperature is equal to the cold fluid inlet temperature, i.e.:

$$\theta=0\quad\text{when}\quad\tau=0$$

- the boundary conditions at two ends of the exchanger for hot stream are:

$$\theta = 0 \quad \text{when} \quad X = 0$$

$$\frac{\partial \theta}{\partial X} = 0 \quad \text{when} \quad X = 1$$

- the boundary conditions at two ends of the exchanger for cold stream are:

$$\frac{\partial \theta}{\partial X} = 0 \quad \text{when} \quad X = 0$$

$$\theta = 0 \quad \text{when} \quad X = 1$$

- The boundary conditions at two ends of the exchanger for the plates are:

$$\frac{\partial \theta_p}{\partial X} = 0 \quad \text{when} \quad X = 0$$

$$\frac{\partial \theta_p}{\partial X} = 0 \quad \text{when} \quad X = 1$$

Results and discussion

In order to validate the numerical results obtained in this paper, they are compared with those predicted in Sharifi *et al.* (1995). To conduct a meaningful comparison, the plate geometry and flow details used in Sharifi *et al.* (1995) are incorporated in the present model. These information are listed in Table I. A sample set of numerical data for water in steady-state condition predicted in Sharifi *et al.* (1995) are shown in Table II. These results were validated by comparison to experimental measurements (Sharifi *et al.*, 1995).

Channel length	L	0.33 m
Channel width	W	0.09 m
Plate heat transfer area	A	0.0297 m^2
Mean channel gap	B	0.005 m
Channel flow area	A_x	4.5×10^{-4} m^2
Plate thickness	Δ	0.0005 m
	Hot fluid	Cold fluid
Inlet temperature	65.0°C	10.0°C
Specific heat	4190 J/kg.K	4180 J/kg.K
Mass flow rate	0.045 kg/s	0.235 kg/s
Reynolds number	1,881	4,263

Table I.
Plate geometry and flow details (Ciofalo *et al.*, 1996)

The numerical results from the present modeling in steady-state condition, when no conduction in plates and in flow channels is present have been presented in Table III. The data in Table III are compared with those in Table II. The comparison is shown in Figure 3, indicating quantitative agreement in channels 1-5. There is a deviation of two sets of data at the beginning of channel 6. The authors, after careful consideration came to the conclusion that the flow temperatures at $x = 0$ and $x = 0.33\,\text{m}$ in channel 6, as copied from Sharifi *et al.* (1995) are incorrect. The above comparison validates the correctness of numerical modeling developed in this paper.

The program can now be used to study the effect of axial conduction in plates as well as in flow channels on temperature distributions, individually and simultaneously. Temperature distributions of flow in the channels of plate heat exchanger have been shown in Table IV when axial conduction both in plates and in flow channels is present. The variations of hot and cold fluid temperatures in channels 1 and 2 as well as the temperature variations of plate 1 (between channels 1 and 2) with respect to channel length are shown in Figure 4. The same temperature distributions have been plotted for channels 5 and 6 and plate 5 in Figure 5.

Temperature distributions of flow in the channels of plate heat exchanger have been shown in Table V, when axial conduction is present in the plates only. Temperature distributions of flow in the channels of plate heat exchanger have been shown in Table VI, when axial conduction is present in the flow channels only. Temperature distributions of flow in the channels of plate heat exchanger listed in Tables V and VI are not much different with those reported in Table III when there is axial conduction neither in plates nor in flow channels.

To investigate the effect of axial conduction on thermal performance of heat exchanger, the dimensionless axial conduction parameter has been used (Shah and Focke, 1988). This parameter is named $\lambda$for plates and λ' for flow channels:

Table II.
Temperature distributions (°C) in plate heat exchanger channels (Sharifi *et al.*, 1995)

X(cm)	Channel 1	Channel 2	Channel 3	Channel 4	Channel 5	Channel 6
0	65.0	15.3	65.0	15.1	65.0	15.0
6.6	61.6	14.2	58.5	13.9	58.5	13.9
13.2	58.2	13.1	52.7	12.8	52.6	11.6
19.8	55.1	12.1	47.4	11.8	47.4	11.0
26.4	52.0	11.0	42.7	10.9	42.7	10.5
33.0	49.0	10.0	38.0	10.0	38.0	10.0

Table III.
Temperature distributions (°C) in plate heat exchanger channels with no axial conduction (present work)

X(cm)	Channel 1	Channel 2	Channel 3	Channel 4	Channel 5	Channel 6
0	65.0	15.6	65.0	15.21	65.0	12.68
6.6	61.58	14.27	58.53	13.89	58.39	11.98
13.2	58.33	13.06	52.83	12.73	52.61	11.38
19.8	55.28	11.95	47.79	11.70	47.53	10.86
26.4	52.36	10.94	43.32	10.80	43.06	10.40
33.0	49.59	10.0	39.33	10.0	39.1	10.0

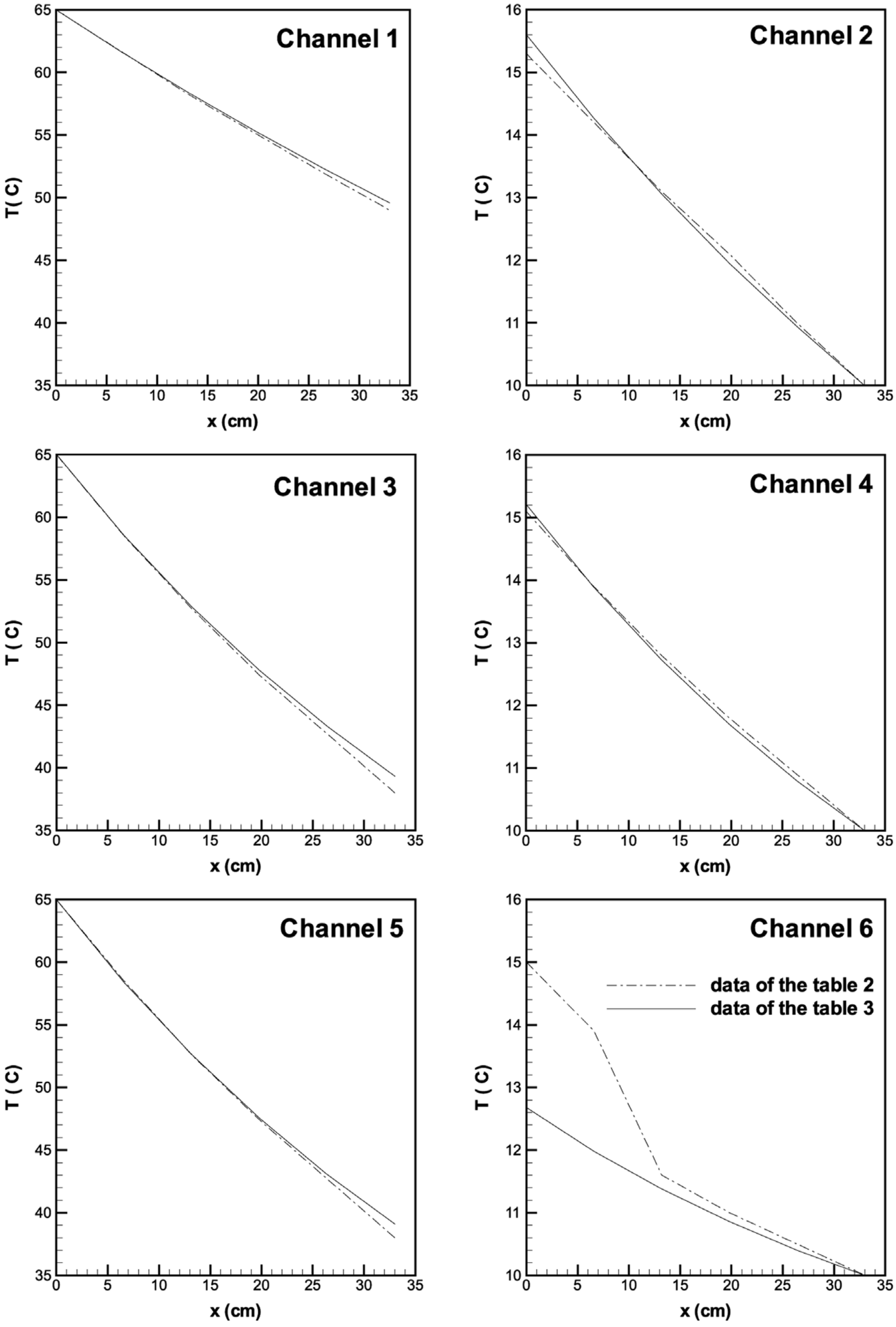


Figure 3.
Temperature distributions in plate heat exchanger channels with no axial conduction, comparing the results of present work with predictions of Sharifi *et al.* (1995) to validate the model

$$\lambda = \frac{k_p A_{xp}}{L C_{min}} \quad \lambda' = \frac{k A_x}{L C_{min}}$$

In which C_{min} is the minimum thermal capacity rate of two fluids. Figures 6-8 show the thermal performance ($\varepsilon - NTU$) curves of the exchanger. The effect of both axial conduction in plates and channels on performance is shown in Figure 6. This figure

Table IV.
Temperature distributions (°C) in plate heat exchanger channels with axial conduction both in plates and flow channels (present work)

X(cm)	Channel 1	Channel 2	Channel 3	Channel 4	Channel 5	Channel 6
0	65.0	15.62	65.0	15.2	65.0	12.67
6.6	61.91	14.15	59.03	13.75	58.9	11.91
13.2	58.65	12.93	53.12	12.58	52.9	11.31
19.8	55.55	11.83	47.90	11.58	47.63	10.80
26.4	52.58	10.83	43.27	10.70	43.01	10.35
33.0	49.46	10.0	38.75	10.0	38.51	10.0

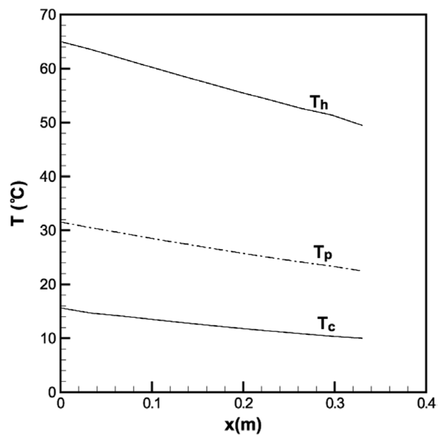


Figure 4.
Temperature distributions of hot and cold streams in channels 1 and 2 and the plate between them

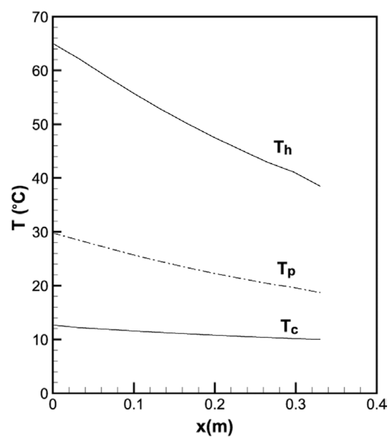


Figure 5.
Temperature distributions of hot and cold streams in channels 5 and 6 and the plate between them

suggests that axial conduction both in plates and channels improves the exchanger performance when $\varepsilon > 0.5$, but does not change the performance when $\varepsilon < 0.5$. The effect of axial conduction in plates alone, on performance is shown in Figure 7. This figure recommends that axial conduction in plates alone, worsens the performance. The effect of axial conduction in flow channels alone, on performance is shown in Figure 8. This figure shows that axial conduction in channels alone, improves the performance. Looking at Figures 7 and 8 show that axial conduction in flow channels has a more severe effect on thermal performance than axial conduction in plates. That means, doubling λ' (from 4.5E-6 to 9.0E-6) increases the effectiveness about 2 percent, while doubling λ (from 5.0E-6 to 10.E-6) decreases the effectiveness only less than 1 percent. Therefore, the reason for increasing the effectiveness with axial conduction

Table V. Temperature distributions (°C) in plate heat exchanger channels with axial conduction in plates only (present work)

x(cm)	Channel 1	Channel 2	Channel 3	Channel 4	Channel 5	Channel 6
0	65.0	15.60	65.0	15.21	65.0	12.68
6.6	61.58	14.27	58.33	13.89	58.39	11.98
13.2	58.33	13.10	52.38	12.73	52.61	11.38
19.8	55.27	11.95	47.80	11.71	47.53	10.86
26.4	52.36	10.94	43.32	10.80	43.06	10.40
33.3	49.59	10.0	39.34	10.0	39.11	10.0

Table VI. Temperature distributions (°C) in plate heat exchanger channels with axial conduction in flow channels only (present work)

X(cm)	Channel 1	Channel 2	Channel 3	Channel 4	Channel 5	Channel 6
0	65.0	15.62	65.0	15.21	65.0	12.67
6.6	61.91	14.15	59.03	13.75	58.90	11.91
13.2	58.65	12.93	53.12	12.58	52.90	11.31
19.8	55.55	11.83	47.90	11.58	47.63	10.80
26.4	52.58	10.83	43.27	10.70	43.00	10.35
33.0	49.46	10.0	38.75	10.0	38.51	10.0

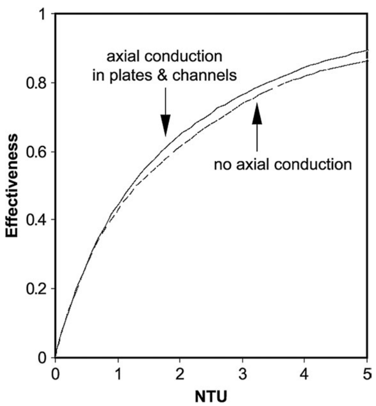


Figure 6. Effect of axial conduction in plates and flow channels on exchange performance

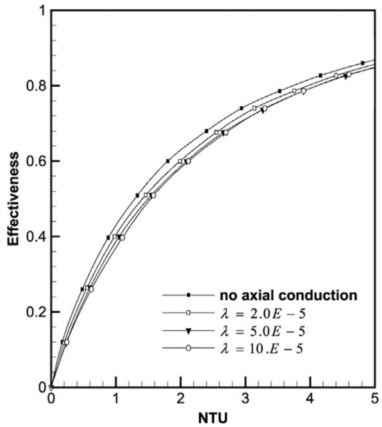


Figure 7.
Effect of axial conduction of plates on exchanger performance

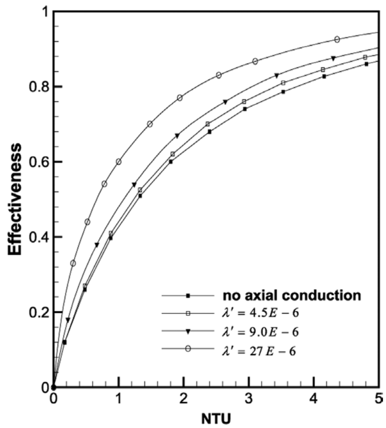


Figure 8.
Effect of axial conduction of flow channels on exchanger performance

both in plates and flow channels in Figure 6 is because of predominance of axial conduction in flow channels.

The variations of overall heat transfer coefficient with respect to hot and cold stream temperatures in channels 3 and 4 are shown in Figures 9 and 10, respectively. These figures predict a linear relationship between the overall heat transfer coefficient and hot and cold stream temperatures. This is because of temperature dependent viscosity in the present analysis, while in a constant viscosity analysis the overall heat transfer coefficient would be constant and the logarithmic mean temperature difference will represent the true temperature difference between two streams.

Since the overall heat transfer coefficient varies along the path of flow, the number of transfer units is defined with respect to the mean overall heat transfer coefficient:

$$\mathrm{NTU} = \frac{\bar{U}A}{C_{\min}} \tag{13}$$

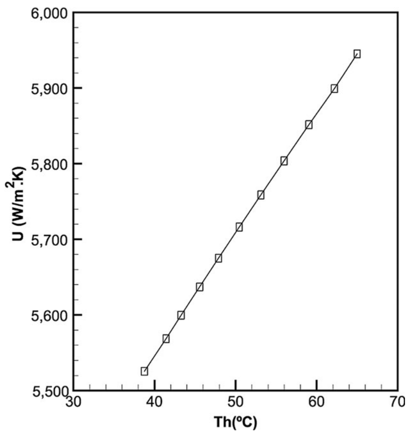


Figure 9.
Variations of overall heat transfer coefficient in channel 3 with respect to hot fluid temperature

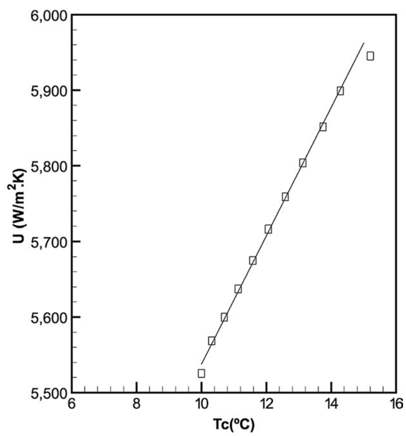


Figure 10.
Variations of overall heat transfer coefficient in channel 4 with respect to cold fluid temperature

in which,

$$\bar{U} = \frac{1}{A}\int_A U \mathrm{d}A \tag{14}$$

The overall heat transfer coefficient based on equation (14) is within 90-95 percent of the overall heat transfer coefficient when viscosity is assumed constant.

The values of coefficient s and exponents q and r in equation (12), incorporated in the model have been taken from (Sharifi *et al.*, 1995) in order to make a reasonable comparison between the two sets of data. It should be mentioned that, heat transfer correlations in plate heat exchangers depend on plate geometry (corrugation angle) and flow regime. A survey of heat transfer correlations in plate heat exchangers is presented in Kumar (1984). The values of s, q and r corresponding to plate type and flow regime may be obtained from (Kumar, 1984) and incorporated in the program to analyze a real plate heat exchanger with corrugated plates. The Reynolds numbers in channels 3 and 4 corresponding to the plate geometry and flow details presented in Table I are 1,881 and 4,263, respectively, making the flow fully turbulent. The equivalent diameter for the flow channel between two flat plates is double the channel gap (2b), and therefore the Reynolds number would be:

$$Re = \frac{\rho u(2b)}{\mu} = \frac{2\dot{m}}{\mu W}$$

where,

$$\dot{m} = \rho A_x u = \rho(bW)u$$

The viscosity in channel 3 is evaluated at the mean temperature of 51.5°C and in channel 4 is evaluated at the mean temperature of 12.55°C. The corresponding values are 5.317×10^{-4} and 1.225×10^{-3} kg/m s, respectively.

Conclusions

A finite difference-based computer program is developed to study the effect of axial conduction in plates and flow channels on thermo-hydraulic characteristics of plate heat exchangers while fluids' viscosities are temperature dependent. The program is run when the dimensionless axial conduction parameters for plate and channels are chosen to be zero and specific plate dimensions and flow details are incorporated. The model is validated by comparing these numerical results with those published in an established reference carried out ignoring the effect of axial conduction and using the same plate geometry and flow details. Quantitative agreement is achieved and the correctness of the model is verified. The program is then run taking into account the effect of axial conduction in plates as well as in flow channels, individually and simultaneously. Temperature distributions are achieved and characteristic curves are plotted. The effect of axial conduction in plates alone on temperature distributions of flow in the channels of plate heat exchanger is not severe when the exchanger effectiveness is less than 0.5. The same effect is observed for axial conduction in flow channels alone. The numerical results show that axial conduction in plates worsens the performance of the exchanger, while axial conduction in flow channels improves the exchanger performance. The results also indicate that the effect of axial conduction in flow channels on exchanger performance is more severe than axial conduction in plates, so that when axial conduction in plates and flow channels are taken into consideration simultaneously, the effectiveness is increased. A linear relationship between the overall heat transfer coefficient and either cold or hot fluid temperature is observed.

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